PAPER

Passive Element Approximation of Equivalent Circuits by the Impedance Expansion Method

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SUMMARY The impedance expansion method (IEM), which was previously proposed by the authors, is a circuit-modeling technique for electrically-very-small devices. The equivalent circuits derived by the IEM include dependent voltage sources proportional to the powers of the frequency. However, the previous report did not describe how circuit simulators could realize such dependent voltage sources. This paper shows how this can be achieved by approximating the equivalent circuit using only passive elements.

key words: *equivalent-circuit modeling, method of moments, impedance expansion method*

1. Introduction

Electrically-very-small devices are widely used for various wireless and wired systems. Typical examples include electrodes for intrabody communications [1], coils for wireless power transfer systems [2], high-frequency transformers, etc. It is well known that undesired resonances or radiations may occur at the usual operating frequencies of these devices, and they affect their operating characteristics or cause noises [3]. Therefore, these devices should be treated as sources of high-frequency electromagnetic fields even if they are much smaller than the wavelength.

The electromagnetic characteristics of electrically-very-small devices may be approximated by equivalent circuits. This approach has the following benefits:

1. small-scale circuit models can be easily analyzed via theoretical approaches, which give us an insight into the operation mechanism of the devices and how to design matching circuits, noise filters, etc.
2. the interaction between the electromagnetic fields and the non-linear electronic circuits can be analyzed only by importing the equivalent-circuit parameters into versatile circuit simulators.

Our previous study proposed a circuit-modeling technique for electrically-very-small devices called the impedance expansion method (IEM) [4], which is based on the Laurent series expansion of the self-/mutual impedances in the method of moments (MoM) [5] with respect to the complex angular frequency s. The IEM can be regarded as an extension of the partial element equivalent circuit (PEEC) method [6], [7], which had been known, and is as accurate as the induced electromotive force (EMF) method.

In the equivalent circuits derived by the IEM, the impedance components proportional to s^{-1} and s are represented by capacitors and inductors, respectively. On the other hand, the higher-degree components are represented by dependent voltage sources. However, the previous report did not describe how circuit simulators could realize such dependent voltage sources, and this was left for further studies. This paper shows how this can be achieved by approximating the equivalent circuit using only passive elements.

This paper is organized as follows. Section 2 describes the self- and the mutual impedances and the equivalent circuit of an example problem, which was also discussed in [4]. Section 3 describes the parameters of the equivalent circuit approximated only by passive elements. Section 4 describes the simulated results of the approximate circuit model by a versatile circuit simulator. Finally, Sect. 5 concludes this paper.

2. Example Problem

This paper deals with the problem that has been discussed in [4]. As shown in Fig. 1, a straight wire with radius a and length $3l$ has feeding ports 1 and 2 at $z = l$ and $2l$, respectively. The current distributions along the wire axis $I(z)$ are expanded by the port currents I_1, I_2 and basis functions $f_1(z), f_2(z)$ such that

$$I(z) = I_1 f_1(z) + I_2 f_2(z), \tag{1}$$

where these basis functions are shown in Fig. 2, and their mathematical definitions are as follows ($m = 1, 2$):

$$f_m(z) = \begin{cases} \dfrac{l - |ml - z|}{l}, & |z - ml| < l \\ 0, & \text{elsewhere} \end{cases}, \tag{2}$$

$$\frac{\partial f_m(z)}{\partial z} = \begin{cases} \dfrac{ml - z}{|ml - z| l}, & |z - ml| < l \\ 0, & \text{elsewhere} \end{cases}. \tag{3}$$

Based on the IEM, the self-/mutual impedance between $f_m(z)$ and $f_n(z)$ are expanded into Laurent series with respect to the complex angular frequency s and approximated by the terms proportional to s^{-1}, s, and s^2, i.e.

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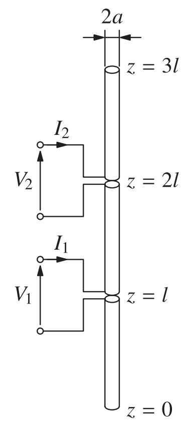

Fig. 1 A straight wire with two feeding ports.

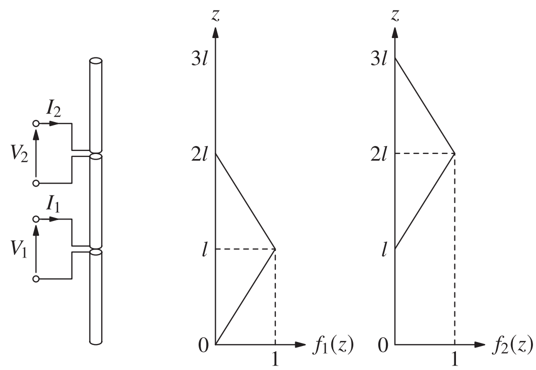

Fig. 2 Piecewise linear basis functions.

$$Z_{mn} = s^{-1} Z_{mn}^{(-1)} + s Z_{mn}^{(1)} + s^2 Z_{mn}^{(2)}. \tag{4}$$

The coefficients for the respective powers are as follows [4]:

$$p_{mn} = \frac{\zeta c}{4\pi} \int_{(m-1)l}^{ml} \int_{(n-1)l}^{nl} \frac{1}{R}\, dz' dz, \tag{5}$$

$$Z_{mn}^{(-1)} = p_{mn} - p_{m(n+1)} - p_{(m+1)n} + p_{(m+1)(n+1)}, \tag{6}$$

$$Z_{mn}^{(1)} = \frac{\zeta}{4\pi c} \int_0^{3l}\!\!\int_0^{3l} f_m(z)\, f_n(z')\, \frac{1}{R}\, dz' dz + \frac{\zeta}{8\pi c} \int_0^{3l}\!\!\int_0^{3l} \frac{\partial f_m(z)}{\partial z} \frac{\partial f_n(z')}{\partial z'} R\, dz' dz, \tag{7}$$

$$Z_{mn}^{(2)} = -\frac{\zeta}{6\pi c^2} \left[\int_0^{3l} f_m(z)\, dz \right] \left[\int_0^{3l} f_n(z')\, dz' \right], \tag{8}$$

where ζ is the wave impedance; c is the speed of light; and R is the distance between the observation and the source points, which is approximately expressed as

$$R \simeq \sqrt{a^2 + (z - z')^2}. \tag{9}$$

The self- and the mutual impedances expressed by Eqs. (4)–(8) can also be represented by the equivalent circuit shown in Fig. 3, in which the parameters are as follows:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}^{-1}, \tag{10}$$

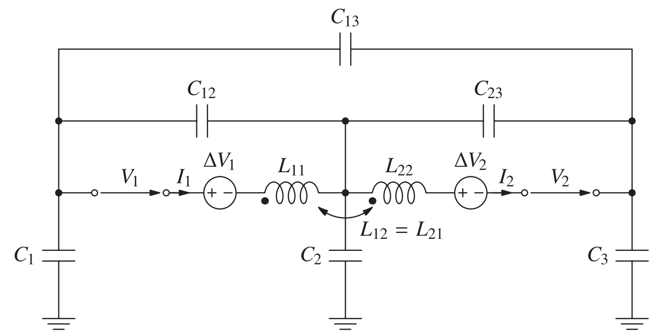

Fig. 3 Equivalent circuit model by the IEM.

$$C_m = \sum_{n=1}^{3} c_{mn}, \quad C_{mn} = -c_{mn}, \tag{11}$$

$$L_{mn} = Z_{mn}^{(1)}, \tag{12}$$

$$\Delta V_m = s^2 Z_{m1}^{(2)} I_1 + s^2 Z_{m2}^{(2)} I_2. \tag{13}$$

By using the circuit parameters, the self- and the mutual impedances can be rewritten as follows:

$$Z_{11} = \frac{p_{11} - p_{12} - p_{21} + p_{22}}{s} + sL_{11} + s^2 Z_{11}^{(2)}, \tag{14}$$

$$Z_{12} = \frac{p_{12} - p_{13} - p_{22} + p_{23}}{s} + sL_{12} + s^2 Z_{12}^{(2)}, \tag{15}$$

$$Z_{22} = \frac{p_{22} - p_{23} - p_{32} + p_{33}}{s} + sL_{22} + s^2 Z_{22}^{(2)}. \tag{16}$$

According to Eq. (8), the following inequalities hold in general:

$$Z_{11}^{(2)} \leq 0, \quad Z_{22}^{(2)} \leq 0, \quad Z_{11}^{(2)} Z_{22}^{(2)} \geq Z_{12}^{(2)2}. \tag{17}$$

If we let $a = 1.5$ mm and $l = 75$ mm, the numerical values of the circuit parameters are as follows:

$$\begin{aligned} &C_1 = C_3 = 933.0174\,\text{fF}, && C_2 = 799.3008\,\text{fF}, \\ &C_{12} = C_{23} = 216.6863\,\text{fF}, && C_{13} = 45.40740\,\text{fF}, \\ &L_{11} = L_{22} = 36.60338\,\text{nH}, && L_{12} = L_{21} = 15.02211\,\text{nH}, \end{aligned}$$
$$Z_{11}^{(2)} = Z_{12}^{(2)} = Z_{21}^{(2)} = Z_{22}^{(2)} = -1.2508654 \times 10^{-18}\,\Omega \cdot \text{s}^2.$$

3. Passive Element Approximation of the Equivalent Circuit

The dependent voltage source ΔV_m ($m = 1, 2$), which is expressed by Eq. (13), is expressed in the time domain as follows:

$$\Delta V_m = Z_{m1}^{(2)} \frac{\partial^2 I_1}{\partial t^2} + Z_{m2}^{(2)} \frac{\partial^2 I_2}{\partial t^2}. \tag{18}$$

Because the above expression includes second-order differentials of the currents, they can be realized only in limited circuit simulators. Even if they can be implemented in circuit simulators, the simulated results are often unacceptable in the transient (time-domain) analysis, according to our experience. However, the electromagnetic problem dealt with in

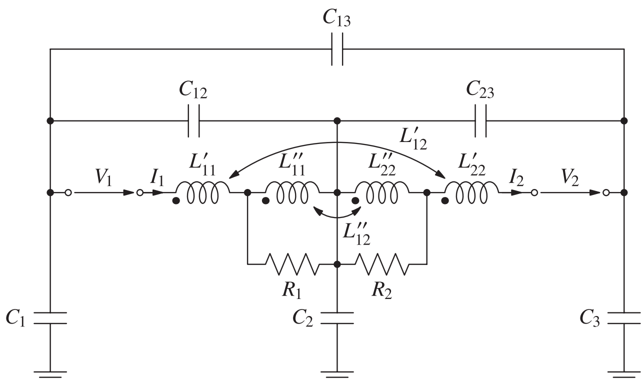

Fig. 4 Equivalent circuit model approximated only by passive elements.

this paper is linear and passive; therefore, its characteristics may potentially be approximated only by passive elements, namely, resistors, inductors, and capacitors. If this is possible, the transient analysis can be run on versatile circuit simulators without special gimmicks. To achieve this purpose, we have to realize a circuit component of which resistance is zero at 0 Hz and increases with the frequency. Among such circuit components, the parallel LR circuit is the simplest one, and its impedance is as follows:

$$\frac{sLR}{R+sL} = sL - s^2\frac{L^2}{R} + \cdots . \tag{19}$$

Based on this idea, this paper proposes the equivalent circuit model shown in Fig. 4.

The self- and the mutual impedances of the approximate equivalent circuit are as follows:

$$Z_{11} = \frac{p_{11}-p_{12}-p_{21}+p_{22}}{s} + sL'_{11} + \frac{sL''_{11} + s^2\dfrac{L''_{11}L''_{22}-L''^2_{12}}{R_2}}{1+s\left(\dfrac{L''_{11}}{R_1}+\dfrac{L''_{22}}{R_2}\right)+s^2\dfrac{L''_{11}L''_{22}-L''^2_{12}}{R_1R_2}}, \tag{20}$$

$$Z_{12} = \frac{p_{12}-p_{13}-p_{22}+p_{23}}{s} + sL'_{12} + \frac{sL''_{12}}{1+s\left(\dfrac{L''_{11}}{R_1}+\dfrac{L''_{22}}{R_2}\right)+s^2\dfrac{L''_{11}L''_{22}-L''^2_{12}}{R_1R_2}}, \tag{21}$$

$$Z_{22} = \frac{p_{22}-p_{23}-p_{32}+p_{33}}{s} + sL'_{22} + \frac{sL''_{11} + s^2\dfrac{L''_{11}L''_{22}-L''^2_{12}}{R_2}}{1+s\left(\dfrac{L''_{11}}{R_1}+\dfrac{L''_{22}}{R_2}\right)+s^2\dfrac{L''_{11}L''_{22}-L''^2_{12}}{R_1R_2}}, \tag{22}$$

where the relation between p_{11}–p_{33} and the capacitances are as defined by Eqs. (10) and (11). By expanding Eqs. (20)–(22) into Laurent series and ignoring the terms of third degree and higher, they are approximated as follows:

$$Z_{11} \simeq \frac{p_{11}-p_{12}-p_{21}+p_{22}}{s} + s(L'_{11}+L''_{11}) - s^2\left(\frac{L''^2_{11}}{R_1}+\frac{L''^2_{12}}{R_2}\right), \tag{23}$$

$$Z_{12} \simeq \frac{p_{12}-p_{13}-p_{22}+p_{23}}{s} + s(L'_{12}+L''_{12}) - s^2\left(\frac{L''_{11}L''_{12}}{R_1}+\frac{L''_{12}L''_{22}}{R_2}\right), \tag{24}$$

$$Z_{22} \simeq \frac{p_{22}-p_{23}-p_{32}+p_{33}}{s} + s(L'_{22}+L''_{22}) - s^2\left(\frac{L''^2_{12}}{R_1}+\frac{L''^2_{22}}{R_2}\right). \tag{25}$$

If Eqs. (23)–(25) are to be equivalent to Eqs. (14)–(16), respectively, the following conditions should be satisfied:

$$L'_{11} + L''_{11} = L_{11}, \tag{26}$$
$$L'_{12} + L''_{12} = L_{12}, \tag{27}$$
$$L'_{22} + L''_{22} = L_{22}, \tag{28}$$
$$\frac{L''^2_{11}}{R_1} + \frac{L''^2_{12}}{R_2} = -Z^{(2)}_{11}, \tag{29}$$
$$\frac{L''_{11}L''_{12}}{R_1} + \frac{L''_{12}L''_{22}}{R_2} = -Z^{(2)}_{12}, \tag{30}$$
$$\frac{L''^2_{12}}{R_1} + \frac{L''^2_{22}}{R_2} = -Z^{(2)}_{22}. \tag{31}$$

At the same time, the following constraints should also be satisfied:

$$L'_{11} > 0, \quad L'_{22} > 0, \quad L'_{11}L'_{22} \geq L'^2_{12}, \tag{32}$$
$$L''_{11} > 0, \quad L''_{22} > 0, \quad L''_{11}L''_{22} \geq L''^2_{12}. \tag{33}$$

In the problem discussed in this paper, the following conditions hold true:

$$L_{11} = L_{22}, \quad L_{12} > 0, \quad Z^{(2)}_{11} = Z^{(2)}_{12} = Z^{(2)}_{22}. \tag{34}$$

In this case, the following parameters satisfy Eqs. (26)–(33):

$$R_1 = R_2 = -\frac{(L_{11}+L_{12})^2}{2Z^{(2)}_{11}} = 1.065339\,\text{k}\Omega, \tag{35}$$
$$L'_{11} = -L'_{12} = L'_{22} = \frac{L_{11}-L_{12}}{2} = 10.79064\,\text{nH}, \tag{36}$$
$$L''_{11} = L''_{12} = L''_{22} = \frac{L_{11}+L_{12}}{2} = 25.81275\,\text{nH}. \tag{37}$$

The Appendix describes the general expressions for the case that Eq. (34) does not hold true. The next section describes the circuit analysis by employing the circuit parameters in Eqs. (A· 7)–(A· 2).

4. Analysis of the Approximate Circuit by SPICE

Subsequently, the frequency characteristics of the self- and the mutual admittances defined by

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \tag{38}$$

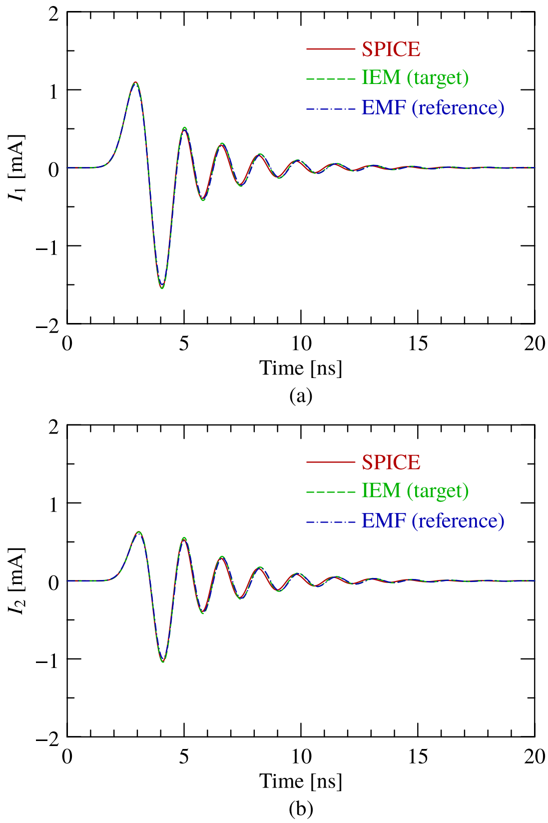

Fig. 5 Transient currents (a) I_1 and (b) I_2.

are discussed by analyzing the equivalent circuit in Fig. 4 with the LTspice (Linear Technology) [8], which is a versatile circuit simulator based on SPICE [9]. Under the condition that port 1 is fed by a voltage source V_1 while port 2 is shorted ($V_2 = 0$), the self- and the mutual admittances can be obtained as follows:

$$Y_{11}(s) = \left.\frac{I_1(s)}{V_1(s)}\right|_{V_2=0}, \quad Y_{21}(s) = \left.\frac{I_2(s)}{V_1(s)}\right|_{V_2=0}, \tag{39}$$

where the frequency-domain currents $I_1(s)$ and $I_2(s)$ are obtained by applying the discrete Fourier transform (DFT) to the transient currents $I_1(t)$ and $I_2(t)$ calculated by the transient analysis. Besides, the excitation voltage $V_1(t)$ has the waveform of Gaussian pulse, i.e.

$$V_1(t) = e^{-\alpha(t-t_0)^2}, \tag{40}$$

where the parameters were selected so that the spectrum at 1 GHz is −60 dB with respect to that at 0 Hz. In addition, the time step in the transient analysis is limited to no longer than 10 ps.

Figure 5 plots the transient currents (a) I_1 and (b) I_2. Here, the lines denoted by "SPICE" indicate the results obtained via the transient analysis by SPICE. On the other hand, the lines denoted by "IEM (target)" indicate the target values based on the original equivalent circuit in Fig. 3. Specifically, they are obtained by applying the inverse DFT to the frequency-domain currents

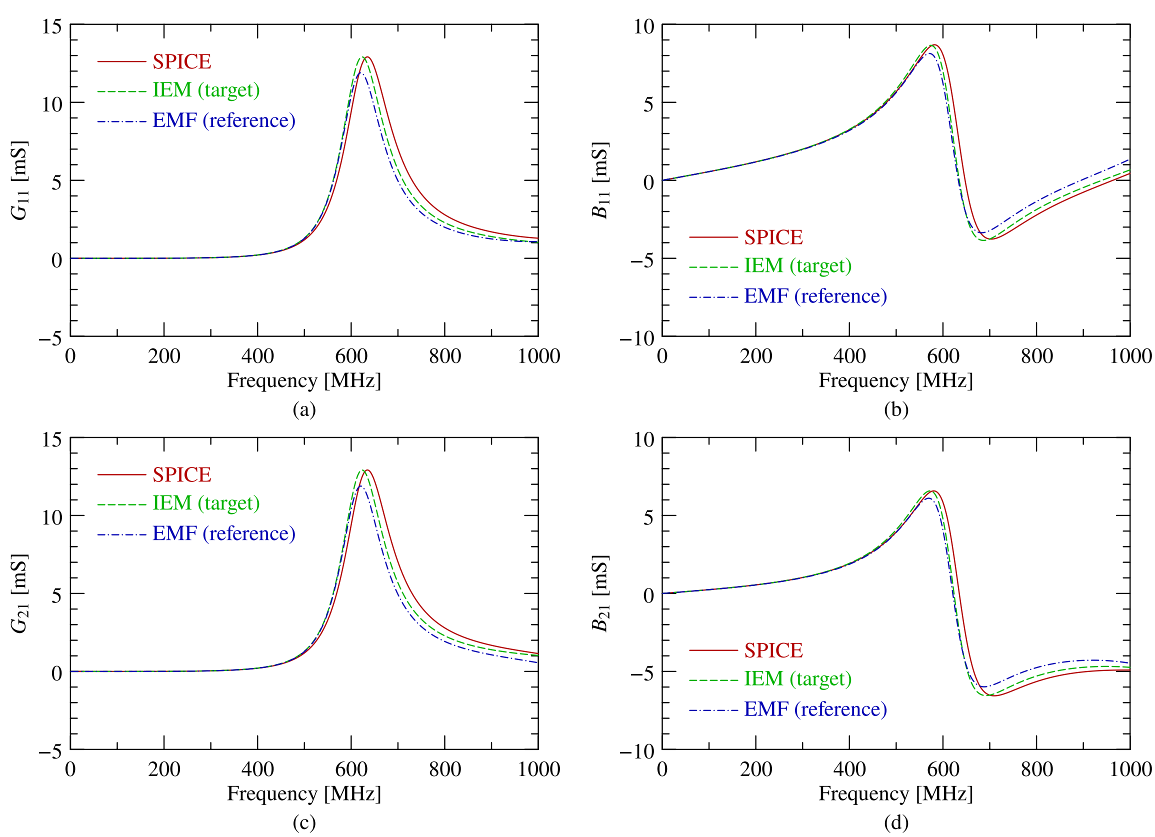

Fig. 6 Self- and mutual admittances: (a) G_{11}, (b) B_{11}, (c) G_{21}, (d) B_{21}.

Table 1 Peak frequency and peak value of G_{11}.

	SPICE	IEM (target)	EMF (ref.)
Peak freq. [MHz]	635	623	621
Peak value [mS]	12.9	12.9	11.9

$$I_1(s) = Y_{11}(s)V_1(s), \quad I_2(s) = Y_{21}(s)V_1(s), \tag{41}$$

where $Y_{11}(s)$ and $Y_{21}(s)$ are obtained by substituting Eqs. (14)–(16) into Eq. (38); and $V_1(s)$ is the Fourier transform of the $V_1(t)$, i.e.

$$V_1(s) = \sqrt{\frac{\pi}{\alpha}} e^{s^2/(4\alpha) - st_0}. \tag{42}$$

In addition, the lines denoted by "EMF (reference)" indicate the reference values based on the full-wave EMF method, which are obtained in a similar manner to the target values except that $Y_{11}(s)$ and $Y_{21}(s)$ were calculated by using numerical integration. The details about the full-wave EMF method were described in [4]. The oscillation period of the currents obtained by SPICE is slightly shorter than those of the target and the reference values. However, their trends almost agree with each other.

Figure 6 plots (a) the real and (b) the imaginary parts of the self-admittance Y_{11} and (c) the real and (d) the imaginary parts of the mutual admittance Y_{21}. Here, the lines denoted by "SPICE" indicate the results by applying the DFT to the currents obtained via the transient analysis and substituting them into Eq. (39). Incidentally, almost the same results can be obtained directly via the AC analysis by SPICE. On the other hand, The lines denoted by "IEM (target)" indicate the target values by substituting Eqs. (14)–(16) into Eq. (38). Also, the lines denoted by "EMF (reference)" indicate the reference values by the full-wave EMF method. In addition, Table 1 summarizes the peak frequency and the peak value of G_{11}. The resonant frequencies of the Y-parameters obtained by SPICE are slightly higher than those of the target and the reference values. According to Table 1, the difference of the result by SPICE from the target value is 1.9% in the peak frequency and that from the reference value is 2.3%. This corresponds to the difference in the oscillation period shown in Fig. 5.

The difference between the results by SPICE and the target values is due to the difference between the approximate expressions for the self- and the mutual impedances in Eqs. (23)–(25) and the exact ones in Eqs. (20)–(22). The results may be improved by adding further elements to the approximate circuit in Fig. 4, and this is left for further studies. In addition, the results by SPICE and the target values reasonably agree with the reference values by the full-wave EMF method. Therefore, the proposed method can be considered to be accurate.

5. Conclusion

Section 2 briefly described the self- and the mutual impedances and the equivalent circuit of the example problem discussed in this paper. Section 3 proposed a technique to approximate the equivalent circuit derived by the IEM only by passive elements. The low-degree terms of the Laurent series expansion of the self- and the mutual impedances of the approximate circuit, namely, the terms proportional to s^{-1}, s, and s^2, are equivalent to those derived via the IEM. Section 4 showed that the approximate circuit model yields reasonable results.

In further studies, practical problems including the IBC and the WPT systems will be investigated. In such problems, three or more ports and more terms in the Laurent series will have to be considered. In these cases, the expressions for the parameters of the approximate circuits will be much more complicated. Therefore, an alternate approach to numerically determine the circuit parameters may be practical. The details are left for further studies.

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Appendix: General Expression for the Parameters of the Approximated Equivalent Circuit

First, Eqs. (29)–(31) should be solved for L''_{11}, L''_{12}, and L''_{22}. By imposing an additional condition

$$\frac{Z_{11}^{(2)}}{R_1} = \frac{Z_{22}^{(2)}}{R_2} \tag{A$\cdot$ 1}$$

and taking Eq. (33) into account, they are obtained as follows:

$$L''_{11} = \sqrt{-\frac{R_1 Z_{11}^{(2)}}{2}\left[1 + \sqrt{1 - \frac{Z_{12}^{(2)2}}{Z_{11}^{(2)} Z_{22}^{(2)}}}\right]}, \tag{A$\cdot$ 2}$$

$$L''_{12} = -Z^{(2)}_{12}\sqrt{-\frac{R_1 Z^{(2)}_{22}}{2Z^{(2)2}_{12}}\left[1-\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right]}, \quad \text{(A· 3)}$$

$$L''_{22} = \sqrt{-\frac{R_2 Z^{(2)}_{22}}{2}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right]}, \quad \text{(A· 4)}$$

Then, the inductances are determined so that Eqs. (26)–(28) are satisfied. Now, L''_{12} is expressed as follows:

$$L''_{12} = \alpha L_{12}. \quad \text{(A· 5)}$$

Because L''_{12} and $Z^{(2)}_{12}$ should be opposite in sign, as expressed by Eq. (A· 3), α should be determined such that $\alpha > 0$ if L_{12} and $Z^{(2)}_{12}$ are opposite in sign, and $\alpha < 0$ if L_{12} and $Z^{(2)}_{12}$ are the same in sign. In other words, the following constraint should be satisfied:

$$\frac{\alpha L_{12}}{Z^{(2)}_{12}} < 0. \quad \text{(A· 6)}$$

By substituting Eq. (A· 5) into Eq. (A· 3), R_1 and R_2 are determined as follows:

$$R_1 = -\frac{2\alpha^2 L^2_{12} Z^{(2)}_{11}}{Z^{(2)2}_{12}}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right], \quad \text{(A· 7)}$$

$$R_2 = -\frac{2\alpha^2 L^2_{12} Z^{(2)}_{22}}{Z^{(2)2}_{12}}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right]. \quad \text{(A· 8)}$$

By substituting Eqs. (A· 7) and (A· 8) into Eqs. (A· 2) and (A· 4) respectively, and taking Eq. (A· 6) into account, L''_{11} and L''_{22} are determined as follows:

$$L''_{11} = \frac{\alpha L_{12} Z^{(2)}_{11}}{Z^{(2)}_{12}}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right], \quad \text{(A· 9)}$$

$$L''_{22} = \frac{\alpha L_{12} Z^{(2)}_{22}}{Z^{(2)}_{12}}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right]. \quad \text{(A· 10)}$$

By substituting Eqs. (A· 9), (A· 5), and (A· 10) into Eqs. (26)–(28) respectively, L'_{11}, L'_{12}, and L'_{22} are determined as follows:

$$L'_{11} = L_{11} - \frac{\alpha L_{12} Z^{(2)}_{11}}{Z^{(2)}_{12}}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right], \quad \text{(A· 11)}$$

$$L'_{12} = (1-\alpha)L_{12}, \quad \text{(A· 12)}$$

$$L'_{22} = L_{22} - \frac{\alpha L_{12} Z^{(2)}_{22}}{Z^{(2)}_{12}}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right]. \quad \text{(A· 13)}$$

By substituting Eqs. (A· 11)–(A· 13) into Eqs. (32), we get the following constraints on α:

$$|\alpha| < -\frac{L_{11} Z^{(2)}_{22}}{\left|L_{12} Z^{(2)}_{12}\right|}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right], \quad \text{(A· 14)}$$

$$|\alpha| < -\frac{L_{22} Z^{(2)}_{11}}{\left|L_{12} Z^{(2)}_{12}\right|}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right], \quad \text{(A· 15)}$$

$$\alpha^2\left\{\frac{Z^{(2)}_{11}Z^{(2)}_{22}}{Z^{(2)2}_{12}}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right]-1\right\}$$
$$-\alpha\left\{\frac{L_{11}Z^{(2)}_{22}+L_{22}Z^{(2)}_{11}}{2L_{12}Z^{(2)}_{12}}\left[1+\sqrt{1-\frac{Z^{(2)2}_{12}}{Z^{(2)}_{11}Z^{(2)}_{22}}}\right]-1\right\}$$
$$+\frac{L_{11}L_{22}-L^2_{12}}{2L^2_{12}} \geq 0. \quad \text{(A· 16)}$$

In particular, if the conditions expressed by Eq. (34) hold, the circuit parameters and the constraints reduce to

$$R_1 = R_2 = -\frac{2\alpha^2 L^2_{12}}{Z^{(2)}_{11}}, \quad \text{(A· 17)}$$

$$L''_{11} = L''_{22} = \alpha L_{12}, \quad \text{(A· 18)}$$

$$L'_{11} = L'_{22} = L_{11} - \alpha L_{12}, \quad \text{(A· 19)}$$

$$|\alpha| \leq \frac{L_{11}+L_{12}}{2L_{12}} \quad \text{(A· 20)}$$

The circuit parameters expressed by Eqs. (A· 7)–(A· 2) are obtained by letting

$$\alpha = \frac{L_{11}+L_{12}}{2L_{12}} \quad \text{(A· 21)}$$

in Eqs. (A· 17)–(A· 19).

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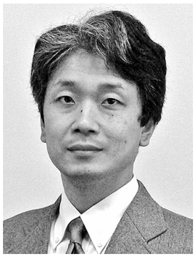

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